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WAKE LABORATORY EXPERIMENT

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1. RESEARCH PROGRAM AND PLAN

The objective of the research program is to obtain analytical and experimental data that can be used for predicting the concentration (as a function of downstream distance) of a passive tracer released into the wake of a self-propelled body travelling through a stably-stratified medium. To achieve this capability, information is required on the wake shape and size. The experimental approach utilizes a stratified flow in which a composite grid is oscillated to produce a steady-state counterpart of the momentumless wake of a self-propelled body. A pH sensitive indicator in which local color changes are generated at the grid by electrical impulses is used for direct observation of the wake development, diffusion and subsequent vertical collapse as the stable stratification counteracts the wake turbulence.

2. EXPERIMENTAL TECHNIQUES

A schematic arrangement of the flow system used for the experiments is given in Figure 1. The tests were performed in a water tank (210 x 10 x 10 cm) in which a stratified flow with a very nearly linear temperature gradient is generated by the mixing of streams of water at different temperatures. The degree of stratification is determined by continually measuring the vertical temperature profile through the use of small thermistor beads that traverse the depth of the water at selected locations at regular intervals.

The velocity of the flow is measured by tracers that are neutrally buoyant at every point in the fluid. This is necessary because the motion of a constant-density dye would be influenced by the density gradient, giving a distorted velocity profile. The technique[1,2] is based on the color change of an indicator solution when the hydrogen ion concentration (pH) is changed at a test point. The solution used consists of water containing 0.01% thymol blue. It is titrated to the end point by adding a small amount of hydrochloric acid and is initially bright orange in color. Fine tungsten wires stretched vertically across the water act as cathodes, and the anode is formed by a

stainless steel wire along the tank bottom. Application of brief voltage pulses (90 volts) to the two electrodes reduces the hydrogen-ion concentration at the surface of the tungsten wire corresponding to a local increase in pH. Columns of dark tracer fluid are formed at the surface of the tungsten wire and move with the fluid to yield vivid velocity profiles. Typical flow speeds are about one cm/sec.

As shown in Figure 1, a wake analogous to that of a self-propelled body is generated in the water channel test section through the use of an oscillating grid approximately one quarter inch in diameter. The grid is pulsed electrically to produce a local pH - color change which traces the wake growth downstream of the grid. Measurements of the wake dimensions in both the horizontal and vertical planes are made at different distances from the grid for both unstratified and stratified flow. In the unstratified case, the wake cross-section at any point is circular. When the flow is stratified, buoyancy causes the wake, which initially grows at the same rate in all directions, to collapse in the vertical direction and flatten out horizontally producing a very wide and thin elliptical or rectangular shape.

The determination of the wake dimensions as a function of wake position and ambient stratification is vital for the prediction of the concentration of a passive tracer released at the body. The research discussed in this report has been directed towards this objective.

3. WAKE GROWTH IN UNSTRATIFIED FLOW

The growth of a wake downstream of a body travelling through a medium in which there is no temperature gradient has been well documented in the literature. Theoretical analyses along with low-speed tests in water and hypersonic tests in ballistic ranges and wind tunnels have established relations for wake growth of the form

$$\frac{\ell_m}{D} = K\left(\frac{C_D x}{D}\right)^n \quad (1)$$

where b_∞ is the wake width (circular), D the diameter of the body, C_D the drag coefficient of the body and x the distance in the wake from the body. For unpropelled bodies, measurements of the turbulent wake growth rates yield values of about unity for K and 1/3 for the exponent n, in good agreement with theoretical predictions.

For the turbulent wakes of bodies with hydrodynamical self-propulsion, theoretical analyses[3] yield values for n of 1/4 for plane flow and 1/5 for axially symmetric flow. At large distances behind the body, the wake may asymptote to a constant size[4], although this has not been verified experimentally.

Various attempts[5-9] to model a submarine wake have been reported in which a two-dimensional unsteady flow is produced through the use of paddle-type mixers in a tank containing quiescent water. In these instances, the time t after mixing is equivalent to the distance in the wake x in Equation (1). It has been observed on the present research program that these measurements can be plotted in the form b_∞/D vs. t/D for the cases in which the water is unstratified to give good correlation of the data, with b_∞/D being proportional to $(t/D)^{1/3}$. The value of 1/3 for the exponent is high and indicates that the mixed regions generated in the two-dimensional unsteady experiments are not strictly equivalent to those at large distances behind a self-propelled body, the wake of which would grow at a much slower rate.

In Figure 2, it is shown that tests reported by Schooley and Stewart[10] using a self-propelled model yield a wake in unstratified water that can be correlated by a relation $\frac{b_\infty}{D} = 1.0\left(\frac{x}{D}\right)^{1/3}$ obtained for hypersonic spheres. The high value of 1/3 for the exponent indicates that the wake growth in this case is determined largely by the effects of body drag and the propeller. Similarly, experiments Naudascher,[4] utilizing a jet-disc model in air to produce a steady-state counterpart of the momentumless wake of a self-propelled body, give an initial growth rate of $\frac{b_\infty}{D} = 15\left(\frac{x}{D}\right)^{2/5}$. This very rapid growth may be due to the influence of the jet for which the exponent n has a value of unity. For distances in the wake greater than about 20 diameters, the wake growth in

Naudascher's experiment can be represented by $\frac{\ell_\infty}{D} \approx 1.2(x/D)^{1/4}$ consistent with the growth rate anticipated for a momentumless wake.

In addition to the interpretation of previous work on marine wake simulation in terms of power law growth rates, measurements have been made of the wake growth downstream of an oscillating grid in a one cm/sec flow of unstratified water. The wake growth observed using the pH-color change tracer is plotted in Figure 2 from which it is observed that the wake diameter can be expressed by the relation $\frac{\ell_\infty}{D} \approx 1.3(x/D)^{1/4}$. The wake diameter reaches a size of roughly four times the grid diameter at one hundred grid diameters downstream. The value 1/4 for the exponent is closer to the value anticipated for the wake growth rate behind a self-propelled vehicle than the value 1/3 observed in other simulation experiments. The measurements indicate that the wake may reach an asymptotic size although further tests are required to clarify this point.

Computer results from the theoretical analysis of Ko[11] are reported in the form of several different curves for Froude numbers, Fr of 0.5, 1.0 and 10.0. It is shown in Figure 2 that these can be expressed by one curve of the form $\frac{\ell_\infty}{D} = 1.25 \ (x/D)^{0.22}$ during the early growth period before the effects of stratification are felt. This relation compares very well with the equation $\frac{\ell_\infty}{D} = 1.3 \ (x/D)^{0.25}$ observed in the present experiments for unstratified flow.

The energy of the turbulence generated in the wake by the oscillating grid can be estimated by using the oft-verified result $u' \approx d\ell_\infty/dt$, where u' is a characteristic turbulent velocity fluctuation, and ℓ_∞ is the width of the wake. For the growth rate observed in the experiments, i.e. $\frac{\ell_\infty}{D} = 1.3(x/D)^{1/4}$ this yields $\frac{u'}{u} = 0.325\,(x/D)^{-3/4}$ for the rate at which the turbulence intensity decays downstream of the oscillating grid. In Figure 3 this relation for u' is shown to agree remarkably well with Naudascher's[4] hot wire anemometer measurements of the turbulence intensities u', v', w' on the axis behind a jet-disc model which is used to simulate the momentumless wake of a

self-propelled body. This agreement confirms the hypothesis that the energy of the turbulence at different positions in the wake can be estimated from measurements of the rate at which the wake dimensions change with time.

The results obtained from the laboratory measurements of wake growth and turbulence intensity, shown in Figures 2 and 3, can be used to estimate the size and energy of the wake of a submarine travelling through an unstratified ocean. The predicted diameter of the wake behind a typical vehicle would increase from 150 feet at two miles downstream to 225 feet at ten miles. In the same distance, the turbulent velocity fluctuations in the wake of a vehicle travelling at 10 knots would decay from approximately 2.5 cm/sec at two miles to 0.5 cm/sec at 10 miles. The wake energy represented by the square of the turbulence velocity plays an important role in determining the collapse of the wake when the flow is stratified.

4. STRATIFIED FLOW-WAKE COLLAPSE TIME

The wake of a body travelling through an environment that is stably stratified can be profoundly influenced by the density gradient. Immediately downstream of the vehicle, the wake, which may contain fluid of nearly constant density, will grow at the same rate in all directions, as in the unstratified flow case, (Figure 2). However, as the turbulent energy of the wake decays with increasing distance from the body (Figure 3), the restoring action of buoyancy begins to inhibit the vertical expansion of the wake and at the same time enhances the horizontal growth. At some point behind the body, the wake reaches a maximum vertical size followed by a collapse as the fluid returns under the action of gravity to the level at which its density is the same as the environment.

Many investigations[5-12], primarily two-dimensional unsteady ones, have been directed toward determining the distance or time from the body to wake collapse. A parameter used to characterize the phenomenon is the Brunt-Vaisala period T defined by Equation (2).

$$T = \frac{2\pi}{\left(\frac{g}{\rho}\frac{\partial\rho}{\partial z}\right)^{1/2}} \tag{2}$$

where $\partial\rho/\partial z$ is the ambient density gradient.

Since T is the period at which a parcel of displaced fluid oscillates about its equilibrium-density position, it would be reasonable to expect that the time to wake collapse t_c would be a function of T. In a recent review by Sundaram[13], apparently conflicting results were presented in an attempt to relate t_c to T. However, it was observed on the present research program that this disagreement among results from different investigators can be attributed to the fact that Van de Watering[5,8] expresses the Brunt-Vaisala period per radian and Schooley[6,7] per cycle. In addition Schooley measures the time to collapse t_c from the start of mixing and Van de Watering from the end of mixing. As shown in Figure 4a, this can seriously influence the results since typical values for T are between 5 and 24 sec/cycle with a typical mixing time of three seconds. When allowance is made for the different definitions of T and zero time in Figure 4a t_c/T from the two-dimensional unsteady experiments is observed to lie between 1/4 and 1/2 at low values for T where the mixing time is equal to or greater than t_c. Since the mixing time becomes a decreasing fraction of the collapse time with increasing T, the ratio t_c/T approaches a value of 1/3 at $T \approx 24$ sec/cycle, the upper limit of the measurements. For the experiments[10] with the self-propelled model in which the mixing time is very small relative to the Brunt-Vaisala period T (which itself is only three seconds), t_c/T in Figure 4a also equals 1/3. Measurements reported[14,15] for towed plates are also shown to give approximately the same value for the ratio t_c/T in Figure 4a.

In the present series of experiments, the wake grows downstream of an oscillating grid that is operated continuously in a flow of thermally-stratified water as depicted in Figure 1. Possible influences of mixing time are eliminated and the collapse of the wake is not constrained by the presence of the mixer in the center as in the two-dimensional unsteady experiments.

Measurements were performed for Brunt-Vaisala periods between 8 and 60 extending the range in T closer to the high values (in the hundreds) measured in the ocean. The results shown in Figure 4b indicate that the collapse time is roughly one-third the Brunt-Vaisala period over the complete range of T. It should be emphasized that the collapse time in all the experiments is only an approximate quantity since the arresting of the vertical wake growth followed by the collapse is a gradual process spread out over a considerable period of time.

5. WAKE DIMENSIONS DURING COLLAPSE

5.1 Analysis

To predict the concentration of a passive tracer introduced in the wake of a self-propelled body travelling through a stratified medium, it is first necessary to establish methods for predicting the size of the wake. As part of the present program of research, a theoretical analysis of wake collapse was carried out, by assuming that in the vicinity of collapse, the turbulent energy in the wake (which makes it grow) is equal to the potential energy (which makes it collapse). For a wake of rectangular cross-section with a linear density gradient, the potential energy can be approximated by $\ell_h \rho(\alpha-\beta)\ell_v^3/12$ and the kinetic energy by $\ell_h \rho (u')^2 \ell_v/2$ where ℓ_v is the vertical height of the wake, ℓ_h is the horizontal width, ρ is the ambient density, u' is the turbulent velocity fluctuation, g is the acceleration due to gravity, $\alpha = \frac{1}{\rho}\frac{\partial \rho}{\partial z}\Big|_a$ is the ambient density gradient and $\beta = \frac{1}{\rho}\frac{\partial \rho}{\partial z}\Big|_w$ is the density gradient in the wake caused by the mixing. If the wake were completely mixed, β would equal zero and the wake would have a constant density. The assumption of a rectangular wake will be shown later to be reasonable. The representation of the potential and kinetic energies by the above expressions should be regarded as giving a parametric rather than an exact dependence. Equating the two approximate equations for the kinetic and potential energies yields the following relation for the vertical height of the wake.

$$\ell_v^2 = \frac{6 (u')^2}{g(\alpha - \beta)} \tag{3}$$

It has been established in Figure 3 for momentumless wakes in unstratified flow that the turbulent intensity u' can be equated to the wake growth rate by $u' \approx \frac{d b_\infty}{dt}$. This relation combined with Equation (1), and taking $x = ut$ where u is the vehicle velocity, yields

$$\frac{u'}{b_\infty} = \frac{n}{t} \qquad (4)$$

Combining Equations (4), (3) and (2) gives

$$\frac{b_v}{b_\infty} \approx \frac{0.40\, n}{\left(\frac{t}{T}\right)\left(1 - \frac{\beta}{\alpha}\right)^{1/2}} \qquad (5)$$

In other words, the ratio of the vertical width b_v of the wake under stratified flow conditions to the width b_∞ when the flow is not stratified can be expressed as a simple function of t/T (the ratio of time $t = x/u$ between the body and the wake location to T the Brunt-Vaisala period). The degree of mixing in the wake enters in the term $\left(1 - \frac{\beta}{\alpha}\right)^{1/2}$.

Justification for the t/T scaling can be obtained by noting that interactions involving flow stratification are often characterized by the Richardson number R_i which can be defined as

$$R_i = \frac{b_\infty^2}{(u')^2} \frac{g}{\rho} \frac{\partial \rho}{\partial z} \qquad (6)$$

The parameter t/T is directly related to R_i since Equation (6) leads to

$$R_i = \left(\frac{2\pi}{n}\frac{t}{T}\right)^2 \qquad (7)$$

The effect of mixing on the time to wake collapse t_c can be determined by taking $\frac{t_{\nu}}{t_{\infty}} = 1.0$ in Equation (5). This yields

$$\frac{t_c}{T} = \frac{0.40\,n}{\left(1 - \frac{\beta}{\alpha}\right)^{1/2}} \qquad (8)$$

In Figure 5, the ratio t_c/T is plotted against the degree of mixing β/α for n equal to 1/3 and 1/5. For the completely mixed case, ($\beta/\alpha = 0$, and $n = 1/4$), $t_c/T \approx 0.10$ whereas for the slightly mixed case, ($\beta/\alpha = 0.9$), $t_c/T \approx 0.32$, which is the value observed in Figure 4 which best correlates the available experimental data on wake collapse. As shown in Figure 5, almost an identical variation in t_c/T with β/α has been observed in the computer results from Ko's[11] theoretical analysis in which the value of $\beta/\alpha = 0.9$ best fit the measurements.

5.2 Measurements of Wake Collapse

In addition to the analysis of marine wake collapse and the correlation of earlier results carried out on the present contract, measurements of wake growth and collapse were obtained for Brunt-Vaisala periods up to 60. This compares with a value of 3 for the self-propelled model experiments[10] and 6 to 24 for the two-dimensional unsteady experiments[6-9] that are influenced by the mixing time and the presence of the mixer at the center of the turbulent region. The present experiment was run continuously with the temperature

gradient allowed to decrease gradually with increasing time so that T varied from about 8 sec/cycle at the start of the experiment to 60 at the end, several hours later. During this period, the wake collapse was observed to move downstream as predicted by the relation $t_c/T \simeq 1/3$.

Measurements of the vertical and horizontal growth of the wake, obtained using the pH - color change technique and the experimental configuration shown in Figure 1, are presented in Figure 6 for a Brunt-Vaisala period of 55. From the grid to an x/D of 15 which corresponds to a value for t/T of 0.15, the vertical ℓ_v and horizontal ℓ_ℓ size of the wake are approximately equal to the diameter ℓ_∞ measured in the unstratified flow case, Figure 2. Further downstream, the vertical size of the wake reaches a maximum of about three grid diameters and then decreases as the wake collapses. At the same time, this causes the horizontal extent of the wake to grow at a faster rate than observed in the unstratified flow case. The vertical size of the wake appears to reach an asymptote of roughly 1.7 times the grid diameter by a time of about one Brunt-Vaisala period.

The cross-sectional area of the wake calculated from the measurements in Figure 6 is shown in Figure 7 as a function of time after wake generation. The wake area A_w is normalized by the area of the grid A_D . For unstratified flow, the circular wake grows according to the relation $\frac{A_w}{A_D} = (1.3)^2\left(\frac{ut}{D}\right)^{1/2}$ obtained from the measurements in Figure 2. For stratification, an assumption must be made concerning the shape of the wake. Ko[11] assumes an elliptical wake. In Figure 7, this assumption is shown to lead to an unrealistic decrease in wake area just after collapse at $t/T = 1/3$. The same behavior is observed when the wake area is calculated for measurements reported by other investigators[5-10]. A rectangular wake will naturally give values for A_w which are too high at collapse but which will likely be realistic at later time. The most probable variation of wake area with time is shown by the dotted line in Figure 7. The initially circular wake starts to collapse in the form of an ellipse but quickly approaches a rectangular cross-section. Ko's[11] assumption of an ellipse after collapse could account in part for the fact that his analysis predicts horizontal wake dimensions somewhat greater than those measured at long times after collapse.

5.3 Correlation of Measurements of Wake Size

In the analysis of wake collapse described in Section 5.1, it was established that the ratio of the vertical wake size to the diameter in unstratified flow, ℓ_v / ℓ_∞ was inversely proportional to t/T, the ratio of the time after wake generation (x/u) to the Brunt-Vaisala period. The present measurement from Figure 6 of wake dimensions during collapse for $T = 55$ as well as those reported by four different investigators[6-10] covering a range of Brunt-Vaisala period from 3 to 24 are correlated well when plotted in Figure 8 in the form ℓ_v / ℓ_∞ against t/T. The results include measurements from self-propelled model tests, two-dimensional unsteady experiments and the present experimental configuration shown in Figure 1.

Curves of ℓ_v/ℓ_∞ vs t/T obtained from Equation 5 are plotted in Figure 8 for different values of β/α, the ratio of the density gradient in the wake to the ambient gradient. Comparison with the experimental data yields a value for the degree of mixing β/α of roughly 0.90 to 0.95 which indicates that the wakes studied in the laboratory are only slightly mixed. In Ko's investigation curves are plotted for $\beta/\alpha = 0.9$ showing wake growth and collapse for Froude numbers between 10 and 1/2 which correspond to Richardson numbers defined by Equation (6) between 10^{-2} and 4.0. Using the present analysis, these curves are nearly collapsed in Figure 8 into one curve falling through the experimental data. As a result, the prediction of wake dimensions during collapse can be considerable simplified using the ℓ_v/ℓ_∞ vs t/T scaling.

The observation that the degree of mixing in the wakes is slight, $\beta/\alpha \simeq .9$ to $.95$, needs to be qualified somewhat. In all the measurements, such as those in Figure 6 and Figure 8, stratification affects the initial growth rate only slightly so that $\ell_v / \ell_\infty \approx 0.9$ to 1.0 up to collapse at $t/T \approx 0.3$. The density gradient inside the wake at this time and not that at $t = 0$ determines the wake collapse. Referring to Figure 7, the wake area at $t/T = 0.3$ is roughly ten times the

area of the body. Initially at $t = 0$, the wake may well have been completely mixed with $\beta/\alpha = 0$. However, with growth, the entrainment of such a large quantity of ambient fluid into the wake would be expected to produce a density gradient at collapse not much different from the ambient gradient, as observed. This point requires further investigation.

As the wake collapses vertically, the horizontal growth rate is enhanced so that the horizontal width b_h becomes greater than the diameter in unstratified flow b_∞. The ratio b_h/b_∞ from the present as well as other investigations[6-11] is correlated quite well in Figure 8 by the parameter t/T and reaches a value of about 1.5 at $t/T = 2$ compared with the vertical size $b_v/b_\infty \approx 1/3$. In other words, the wake width at this point is roughly five times the wake height. Ko's theory is shown in Figure 8 to overpredict the horizontal wake size for t/T greater than about 0.7. This is probably due to Ko's assumption of an elliptical wake, although the presence of the sidewalls may have restricted the horizontal growth in some of the experiments.

5.4 Prediction of Wake Dimensions during Collapse

Combining Equation (5) with the measured growth rate in unstratified flow $\frac{b_\infty}{D} = K(x/D)^n$ yields a relation for predicting the vertical extent of a wake during collapse.

$$\frac{b_v}{uT} = \frac{0.40\,Kn}{\left(1 - \frac{\beta}{\alpha}\right)^{1/2}} \left(\frac{x}{D}\right)^{n-1} \qquad (9)$$

In Figure 9, b_v/uT is plotted against x/D. For comparison, Ko's[11] theoretical curves from wake collapse on downstream for Froude numbers of 0.5, 1.0 and 10 are also given in the figure. When plotted in this fashion, Ko's results for different Froude numbers fall into one curve given by $b_v/uT = 0.57\,(x/D)^{-4/5}$. For the present analysis, as shown in Figure 9, this corresponds to Equation (9) with $K = 1.3$, $\beta/\alpha = 0.94$, and $n = 1/5$. A simple scaling law for the post collapse behavior of the wake is thus obtained from the two analyses when the vertical size of

the wake is normalized by uT. Some uncertainty exists over the values to be used for K and n while the degree of mixing in the wake β/α needs to be investigated.

In Figure 10, a comparison is made between predictions of the vertical dimensions of the wake using Ko's[11] computer analysis and the power law growth rates developed on the present program. The vertical size ℓ_v/D is plotted against wake position x/D for $uT/D = 250$ The parameter UT/D is inversely proportional to the square root of the Richardson number. For typical ocean conditions and submarine dimensions, a value of 250 for UT/D corresponds to a submarine speed of roughly five knots.

The curve for unstratified flow, $\ell_\infty/D = K\,(x/D)^n$ plotted in Figure 10 gives the extent of the wake up to collapse. A family of curves obtained from Figure 9 for different degrees of mixing β/α gives the size of the wake after collapse. Comparing the $\beta/\alpha = 0.94$ curve with Ko's[11] analysis, shown by the dotted line in Figure 10, excellent agreement is obtained except in the immediate vicinity of collapse. At the intersection of the unstratified flow and post collapse curves, a maximum difference of roughly 12% is observed. Clearly, a fairing of the two power-law curves would give closer agreement with Ko's result in this region.

6. APPLICABILITY TO SUBMARINE VEHICLES

In the past, turbulent wake growth rates measured in ballistic ranges and in small scale laboratory investigations of unstratified flow over fixed models have been used successfully to predict the wake growth for full-scale unpowered reentry vehicles. In a similar manner, the laboratory measurements of grid generated turbulence can be used to predict the wake growth behind a self-propelled submarine vehicle using Equation (1) up to collapse $(t_c/T \approx 1/3)$ and Equation (9) downstream of collapse. Using the best values presently available for the uncertain parameters in those equations $(K = 1.3,\ n = 1/4,\ \frac{\beta}{\alpha} = 0.93)$, it is possible to estimate submarine wake dimensions for different speeds, diameters and ambient stratification. In addition, the effect of varying the parameters K, n, and $\frac{\beta}{\alpha}$ determined from theory and experiment can be calculated.

Up to collapse, the wake is taken to be circular with

$$\frac{b_\infty}{D} = 1.3\left(\frac{ut}{D}\right)^{1/4} \tag{10}$$

To estimate the vertical extent of a wake during collapse using the power-law analysis, values for $K = 1.3$, $n = 1/4$, and $\beta/\alpha = .98$, are substituted into Equation (9) to give

$$\frac{b_v}{uT} = 0.50\,(x/D)^{-3/4} \tag{11}$$

Noting that $x = ut$ and normalizing by the diameter D gives

$$\frac{b_v}{D} = 0.50T\left(\frac{u}{D}\right)^{1/4} t^{-3/4} \tag{12}$$

To predict the horizontal extent of a wake during collapse, use can be made of the correlation of measurements in Figure 8. The data can be fitted by

$$\frac{b_h}{b_\infty} \approx 1.55\left(\frac{t}{T}\right)^{1/4} \tag{13}$$

Combining this with the unstratified growth rate, Equation (10) gives

$$\frac{b_h}{D} \approx 2.0\left(\frac{U}{TD}\right)^{1/4} t^{1/2} \tag{14}$$

Referring to Equations (12) and (14), it is observed that both the vertical, b_v, and horizontal, b_h, dimensions of the wake vary weakly with the submarine velocity u being proportional to $u^{1/4}$. As a consequence, measurements of wake dimensions at the same time after submarine passage would not be very sensitive to the submarine velocity. As expected, the vertical extent of the wake is very sensitive to the ambient stratification being directly proportional to the Brunt-Vaisala period T. Note that the vertical size of the wake decreases with $t^{-3/4}$ while the horizontal size increases with $t^{1/2}$. At first glance, this would indicate a decrease in wake area with time during collapse. However, as shown in Figure 7, the wake cross-section is probably changing from elliptical to

rectangular during this time so that, in fact, the wake area is increasing. In addition, in Figure 8 it can be seen that ℓ_v/ℓ_∞ breaks away from the $(t/T)^{-1}$ decay rate to a more gradual fall of the form $\frac{\ell_v}{\ell_\infty} \approx .4\left(\frac{t}{T}\right)^{-1/4}$. Combining this with Equation (10) yields

$$\frac{\ell_f}{D} \approx 0.52\left(\frac{uT}{D}\right)^{1/4} \tag{15}$$

Equation (15), which requires further verification, gives a relation for the asymptotic vertical wake size ℓ_f. For the conditions of the present experiment, $u = 1.1$ cm/sec, $D = 0.6$ cm, and $T = 55$ sec., $\frac{UT}{D} = 100$. As shown in Figure 11, Equation (15) gives an asymptotic wake height ℓ_f/D of 1.6 compared with a measured value of 1.7 in Figure 6. For Schooley and Stewart's[10] self-propelled model experiments in which $u = 45$ cm/sec, $D = 2.2$ cm, and $T = 2.8$ sec, equation (15) gives a value of 1.4 for ℓ_f/D equal to that measured.

A similar relation can be obtained for the maximum vertical extent ℓ_m of the wake by substituting $t_c/T \approx 1/3$ from Figure 4 into equation (10) and noting from Figure 8 that $\ell_m/\ell_\infty \approx .9$ to give

$$\frac{\ell_m}{D} \approx 0.9\left(\frac{UT}{D}\right)^{1/4} \tag{16}$$

For the present experiment, equation (16) as shown in Figure 11 gives $\ell_m/D \approx 2.8$ compared with a measured value of 2.9 while for Schooley and Stewart's tests the measured and estimated values are both 2.4.

Ko's[11] analysis can be shown in Figure 11 to give the same variation of ℓ_m/D with UT/D although the values of ℓ_m/D are slightly lower then those observed. In addition, Ko's analysis predicts values of ℓ_m/D less than those measured in the region where an asymptote appears to be reached. This could account in part for Ko's analysis overpredicting the horizontal extent of the wake. Van de Watering et al[8] report measurements of ℓ_m/D and ℓ_f/D

in terms of the initial wake growth rate. Use of Ko's assumptions that $u'/u = 0.25$ initally for Van de Watering's measurements gives values of UT/D between 2 and 10, which would correspond to submarine velocities of less than one knot. Van de Watering's measurements also yield the $(UT/D)^{1/4}$ variation given in Figure 11 in which his measurements at the highest value of $UT/D = 10$ are shown to agree well with Equations (15) and (16).

As shown in Figure 12, equations 10, 12, 14-16 can be used for preliminary estimates of the growth of the wake behind a submarine travelling through a stratified ocean. Parameters chosen for the calculation are a submarine velocity U of 12 knots, a diameter D of 25 feet and a Brunt Vaisala period T of 12 minutes. Referring to Figure 12, the wake grows initially in a conical manner and reaches a diameter of about 100 feet at two minutes. A maximum vertical height of about 110 feet is reached between three and five minutes followed by a collapse to the asymptotic vertical height of approximately 60 feet at fifteen minutes. During this collapse phase, the horizontal growth rate is enhanced so that by fifteen minutes the wake is roughly 260 feet wide. After collapse, the horizontal width would continue to increase but at a slower rate than during collapse. At the present time, the width is taken to grow as $t^{1/4}$, the unstratified flow case. This is probably too high because of lack of vertical entrainment but further investigation is required.

By forty minutes after wake generation, which corresponds to a position nine miles downstream of the submarine, the wake is estimated to be roughly 300 feet wide and 60 feet high. From Figure 3, the turbulent velocity in the wake at this time is estimated to be of order 0.5 cm/sec so that subsequent dispersion of the wake constituents would depend on the background ocean turbulence.

Estimated extremes in horizontal and vertical sizes can be calculated from the equations developed. For a submarine moving very slowly ($u = 2$ knots) through a highly stratified ocean ($T = 5$ min.), the wake height would be 30 feet. In contrast, a fast submarine ($u = 25$ knots in a slightly stratified ocean ($T = 20$ min) would produce a wake 90 feet high. A wake 450 feet wide would be produced for $u = 25$ knots and $T = 5$ min. whereas the width would

be only 150 feet for $U \approx 2$ knots and $T = 20$ min.

With an estimate of the size and shape of the wake available, preliminary predictions of the concentration of a passive tracer introduced at the submarine could be made. It would be reasonable to anticipate maximum concentrations on a vertical line at the wake center and some form of exponential decay in the horizontal direction to zero at the wake edge with little variation in the vertical direction. Quite high values of tracer concentration would conceivably be observed at large distances from the submarine.

7. SUMMARY

During the reporting period, experiments have been performed in which a grid is oscillated in a stably-stratified flow to produce the steady-state counterpart of the momentumless wake of a self-propelled submarine vehicle. A pH sensitive indicator has been used as a neutrally buoyant tracer to visualize wake development and subsequent vertical collapse as the wake turbulence is overcome by the buoyancy gradient.

The wake growth before and after collapse and the distance to collapse have been correlated by using power laws previously applied for the wakes of re-entry vehicles and a theoretical analysis of marine wake collapse developed under the present program. The theory is based on the balance, at wake collapse, between the potential energy of the mixed fluid and the energy of the turbulent velocity fluctuations in the wake. Experimental data available from the literature, as well as that obtained in the present program, have been used in the correlation. In its most fundamental form, the correlation gives the ratio of the vertical extent of the wake in stratified flow to that in unstratified flow as a function of the ratio of the time in the wake (measured from the body) to the Brunt - Vaisala period. Although enhancement of the horizontal wake growth due to vertical collapse has been observed, the area of the wake is considerably less than that for unstratified flow. From the measurements and analysis, equations have been developed for predicting submarine wake dimensions as a function of submarine size and velocity, ambient stratification, and time after submarine passage. The estimated submarine wake dimensions are of the correct magnitude although the equation

should be regarded as preliminary since further investigation is required in several areas. The very flat and wide wakes observed due to stratification lead to the estimate that the concentration of a passive tracer introduced into a submarine wake would remain high for long times after submarine passage.

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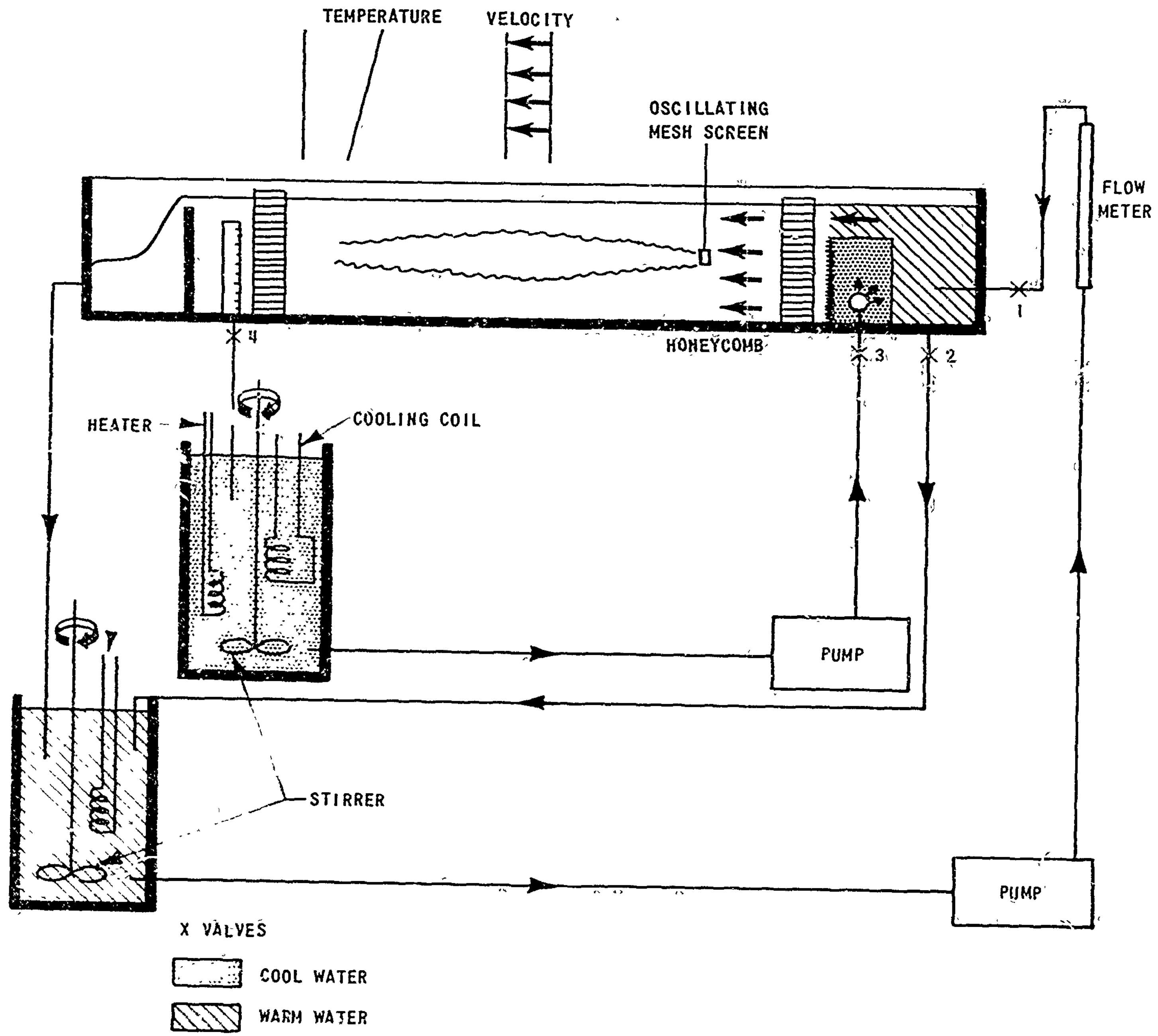

Figure 1 SCHEMATIC ARRANGEMENT OF FLOW SYSTEM

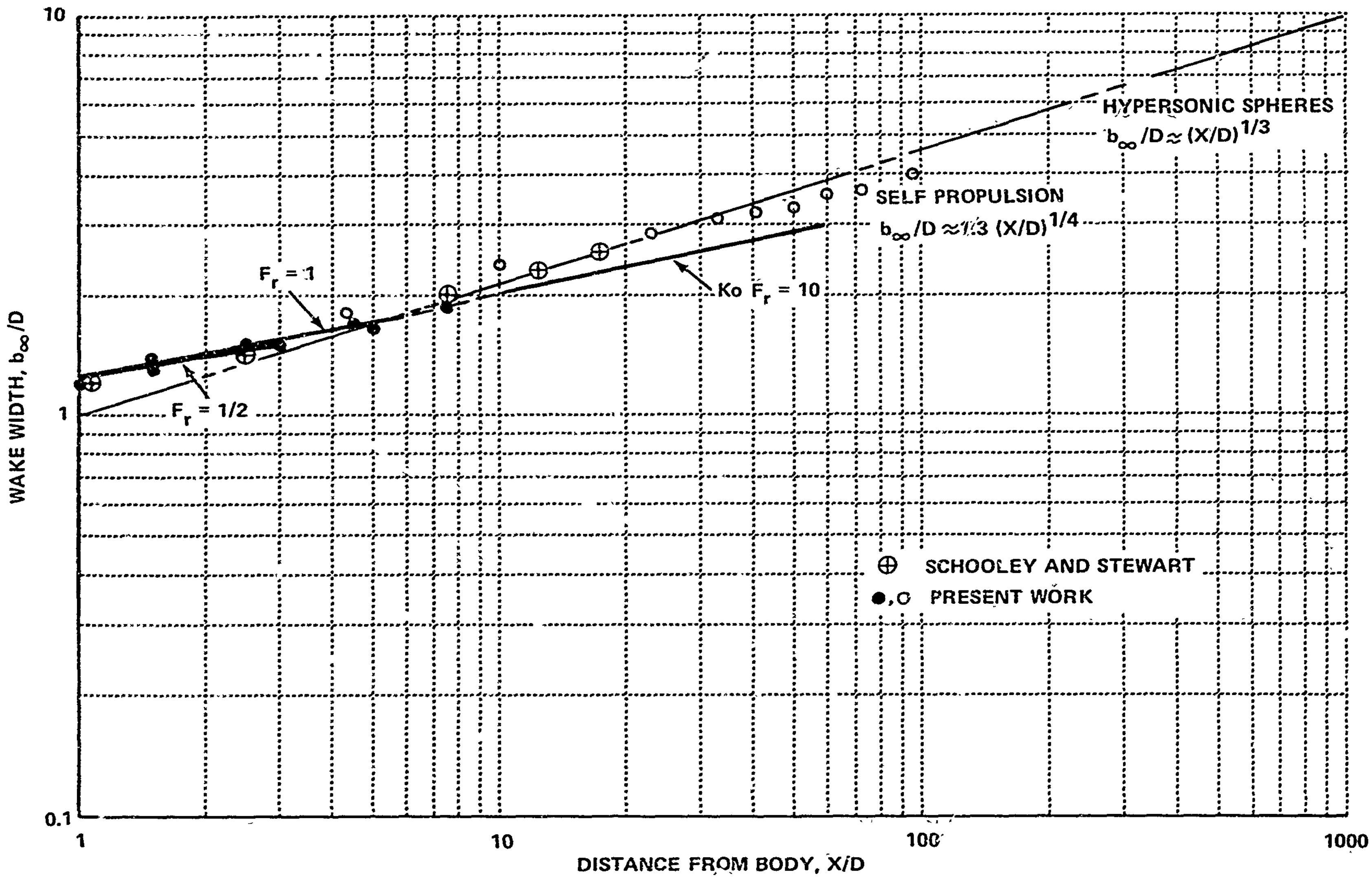

Figure 2 GROWTH RATES IN UNSTRATIFIED FLOW

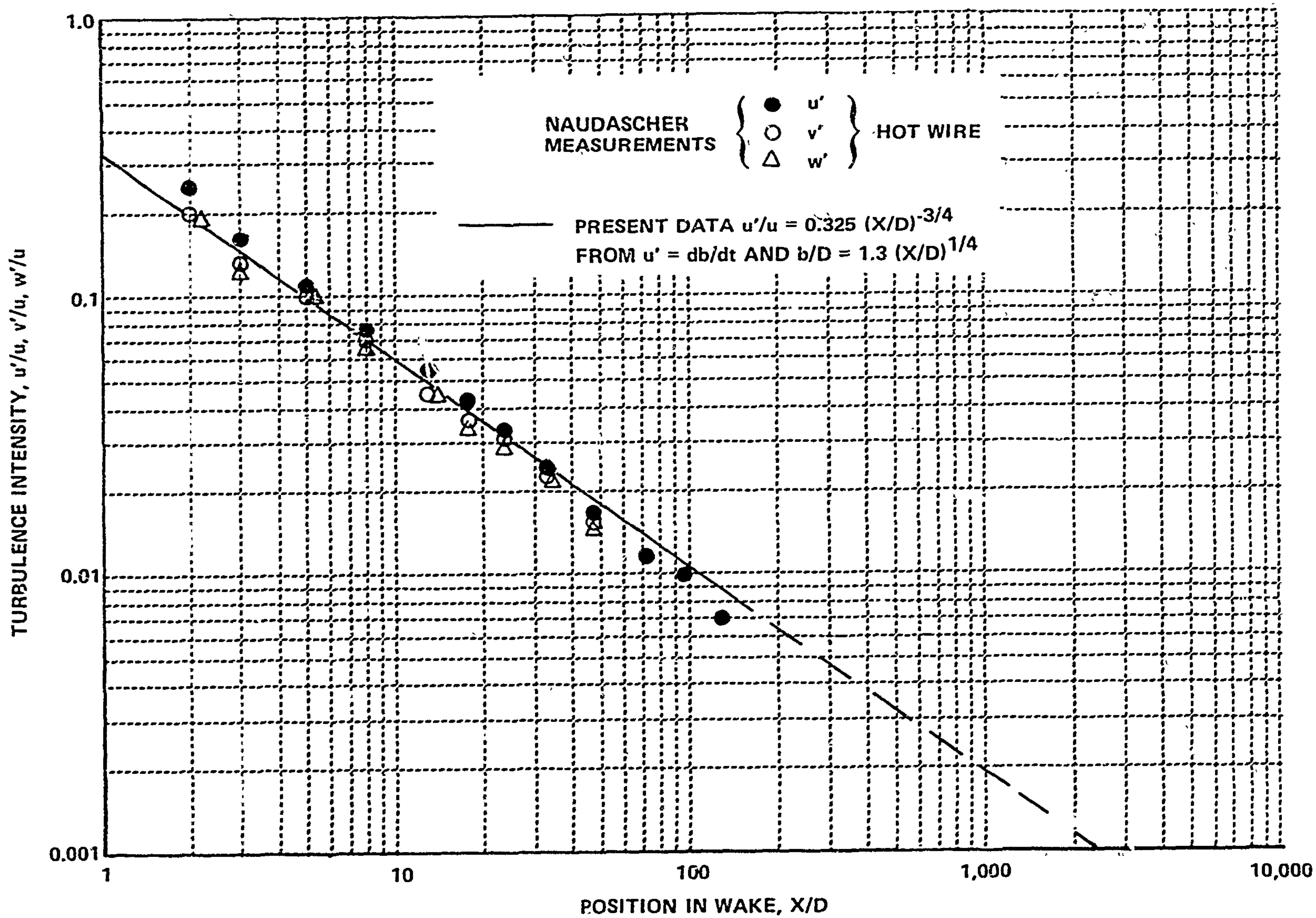

Figure 3 DECAY OF TURBULENCE INTENSITY (UNSTRATIFIED FLOW)

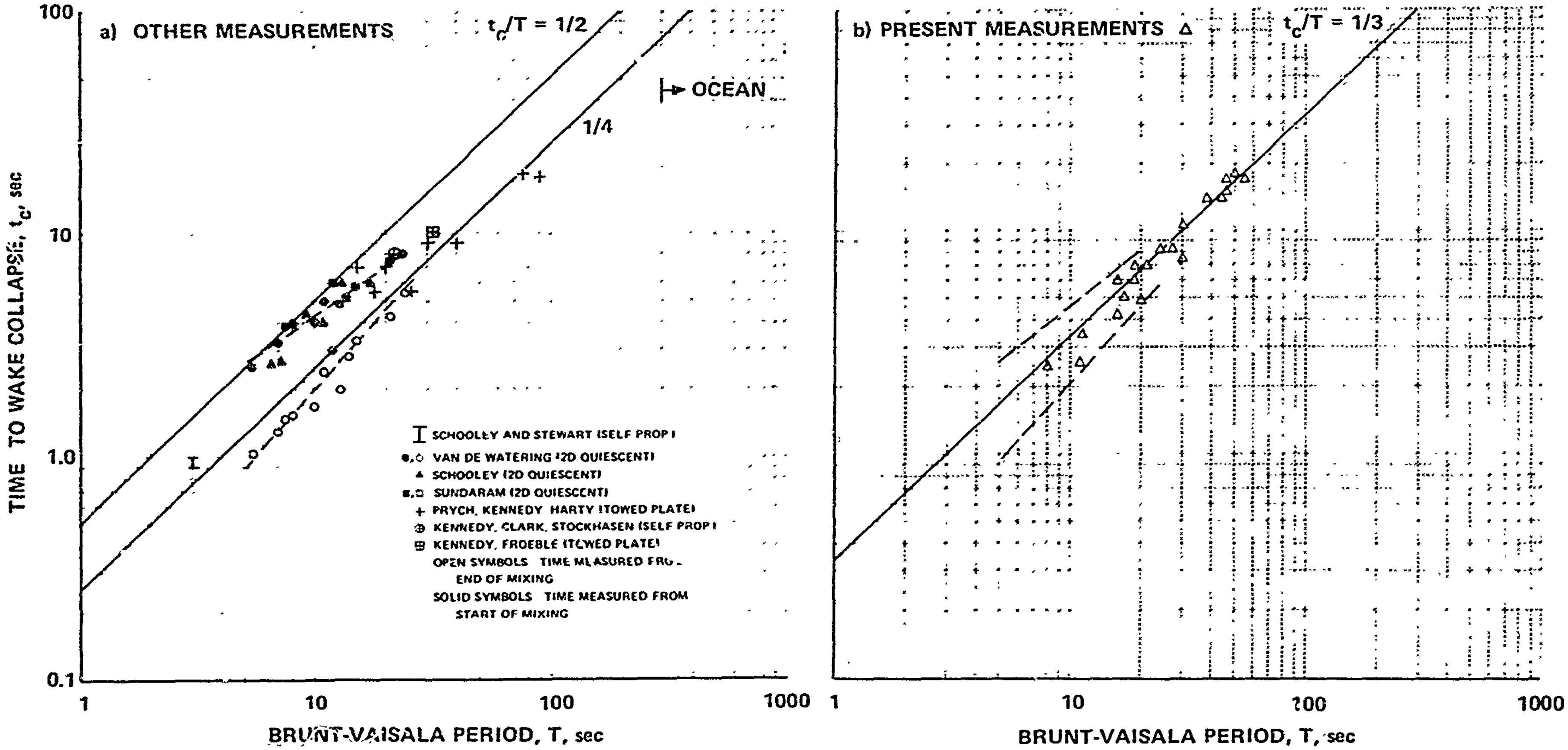

Figure 4 CORRELATION OF WAKE COLLAPSE TIME WITH BRUNT-VAISALA PERIOD

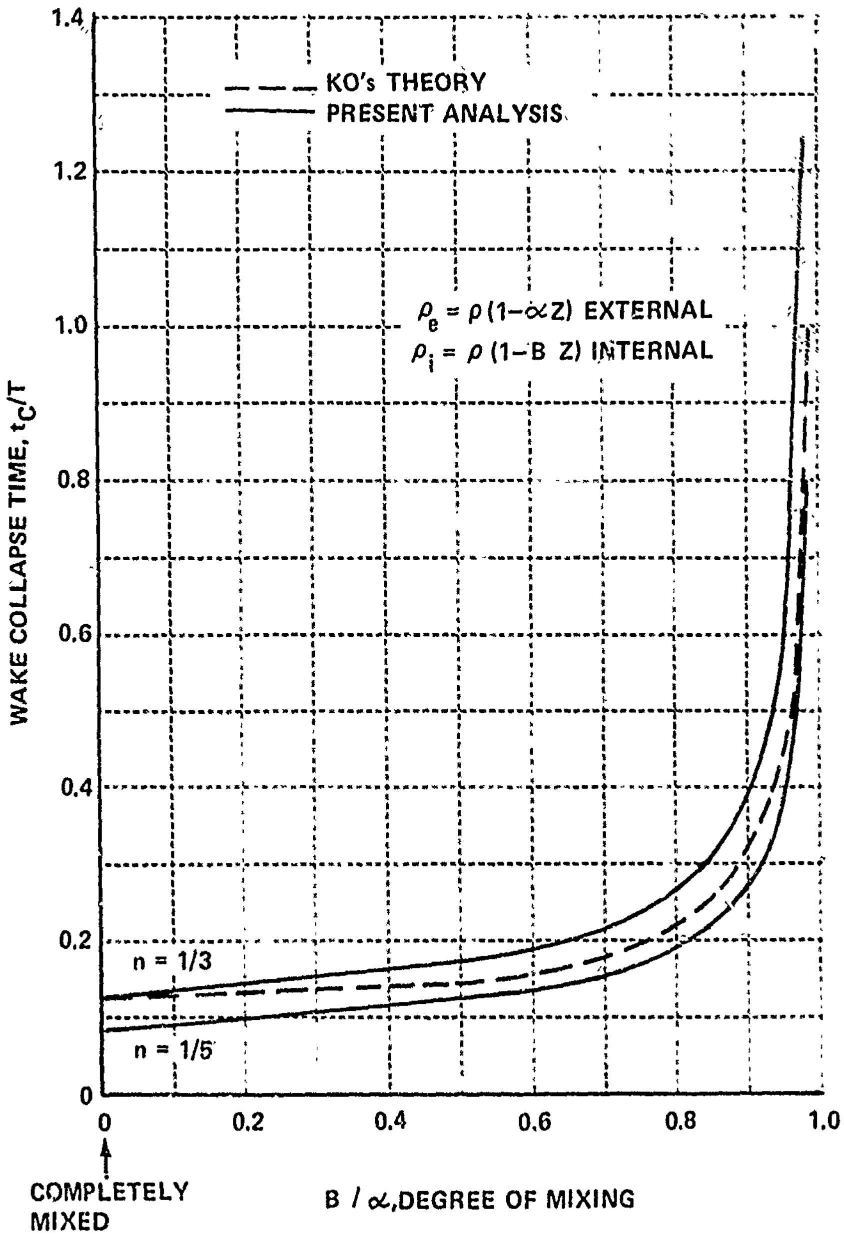

Figure 5 EFFECT OF MIXING ON WAKE COLLAPSE TIME

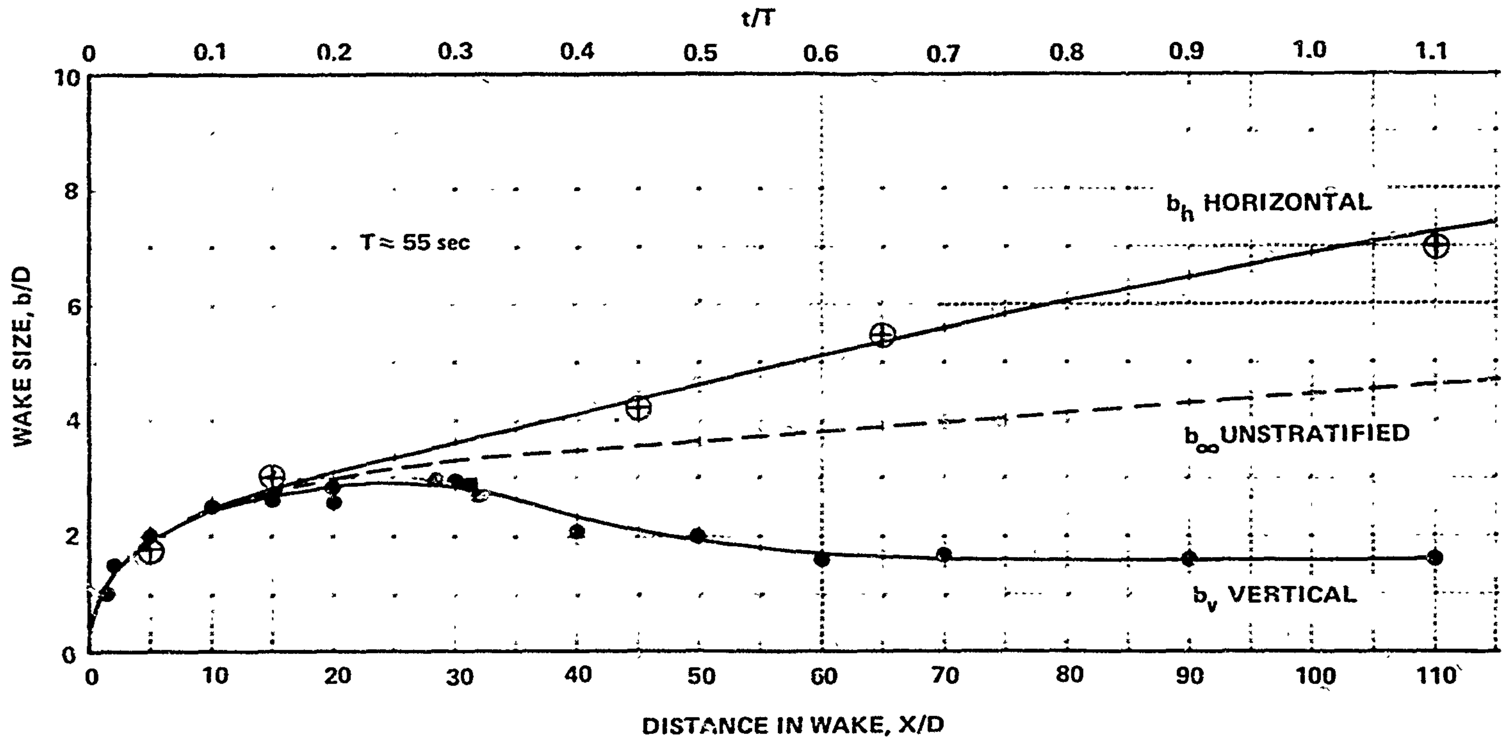

Figure 6 MEASURED WAKE GROWTH

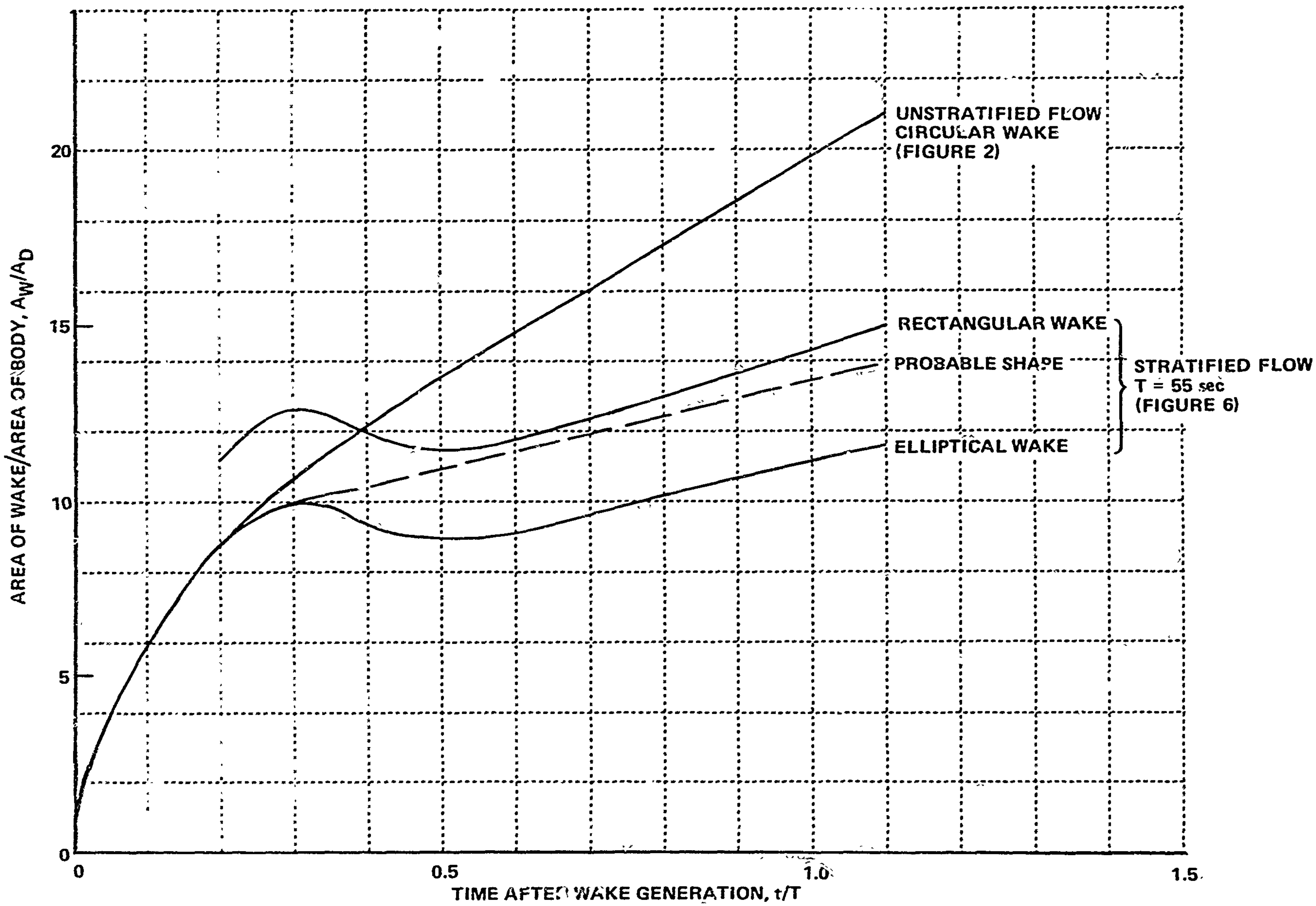

Figure 7 GROWTH OF WAKE AREA

36

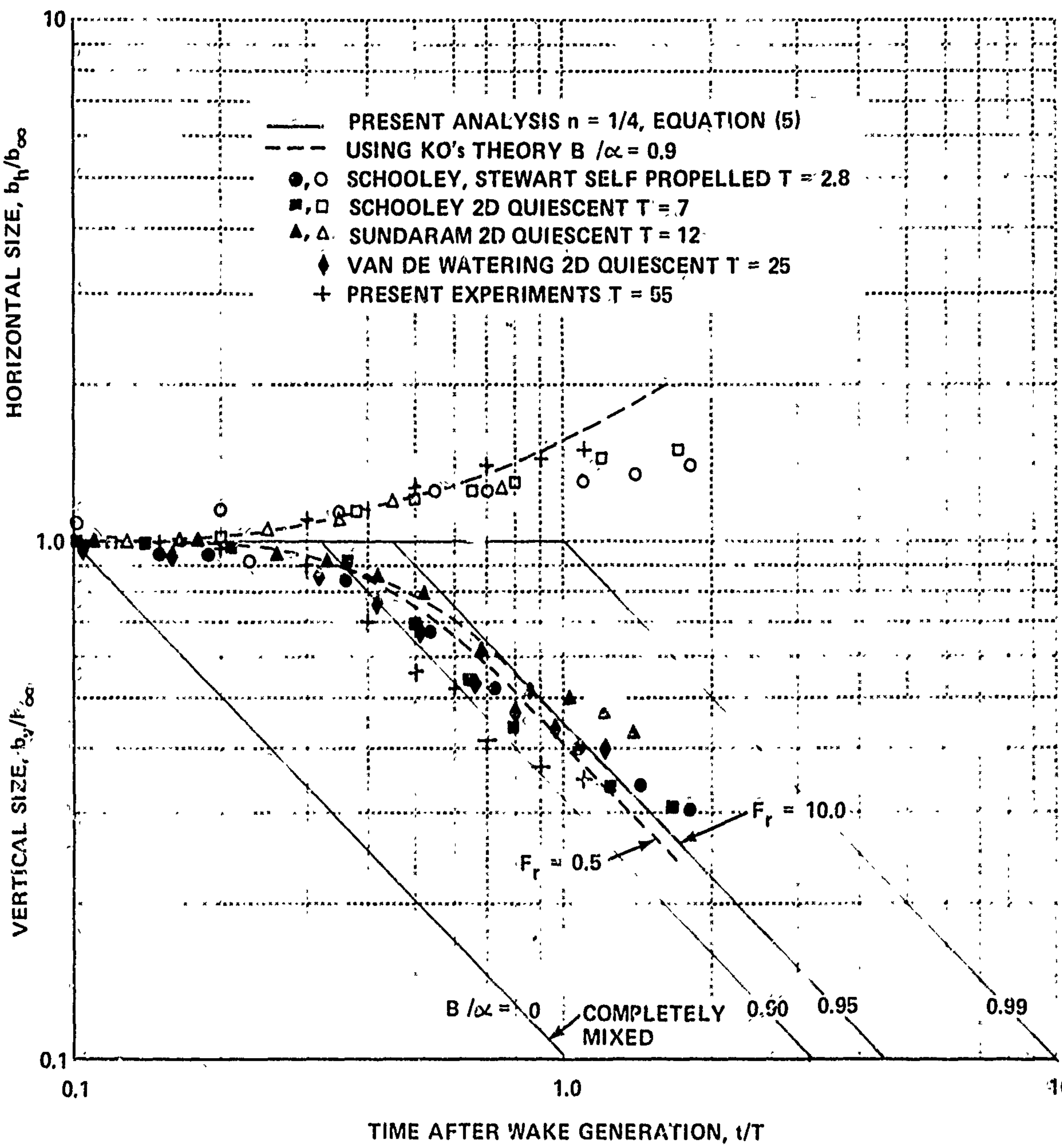

Figure 8 CORRELATION OF WAKE DIMENSIONS DURING COLLAPSE

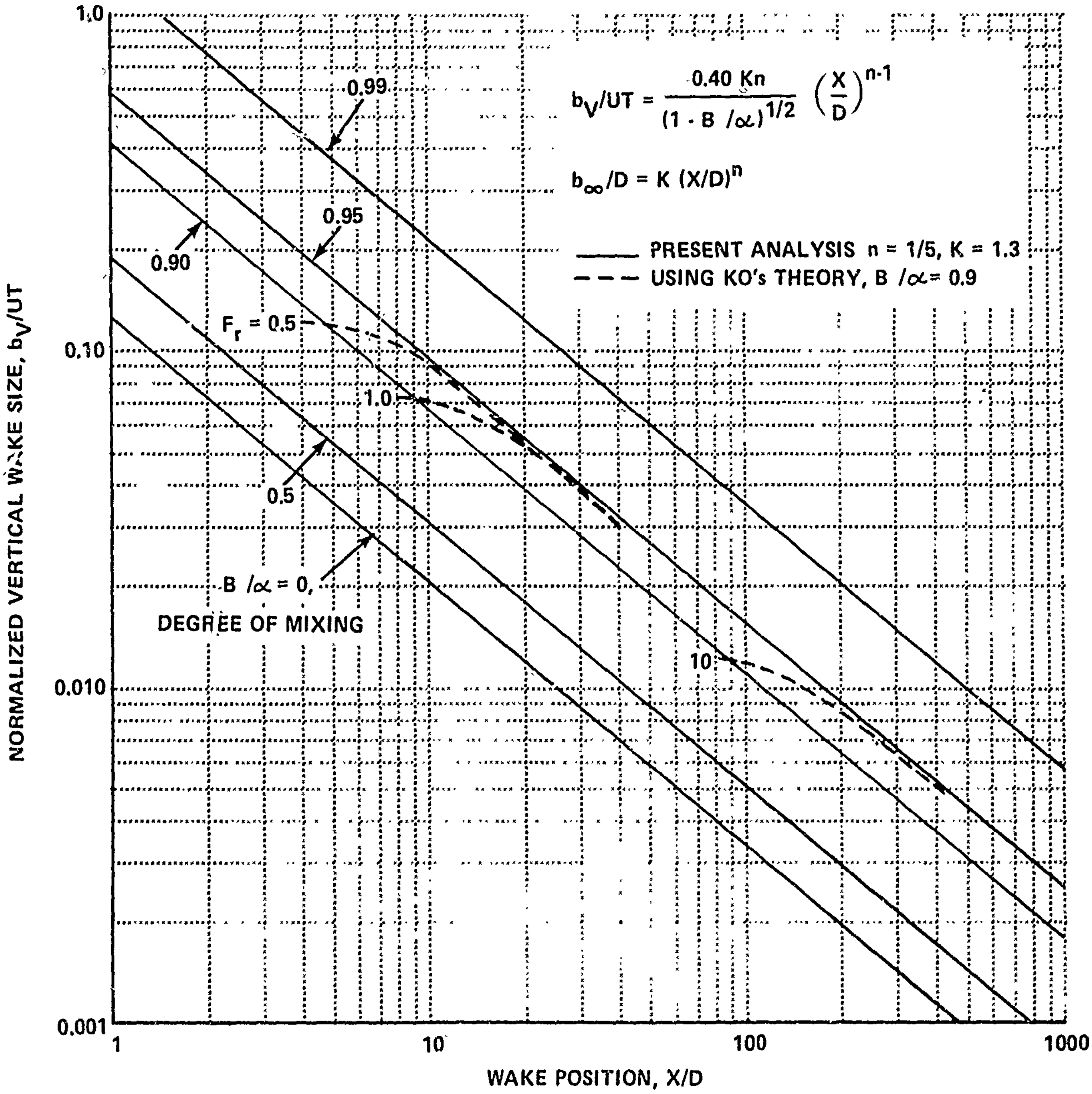

Figure 9 SCALING LAWS FOR VERTICAL SIZE DURING COLLAPSE

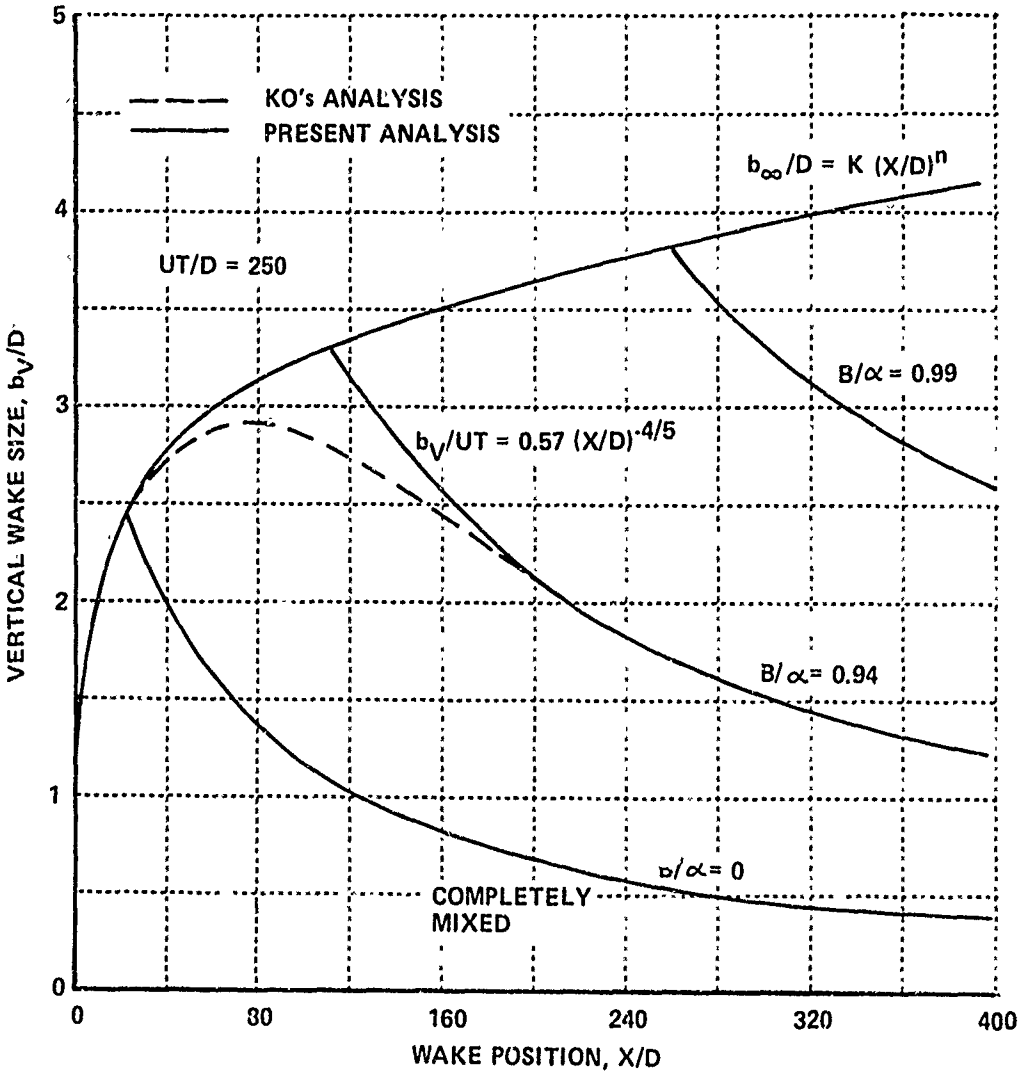

Figure 10 PREDICTIONS OF VERTICAL WAKE SIZE, UT/D = 250

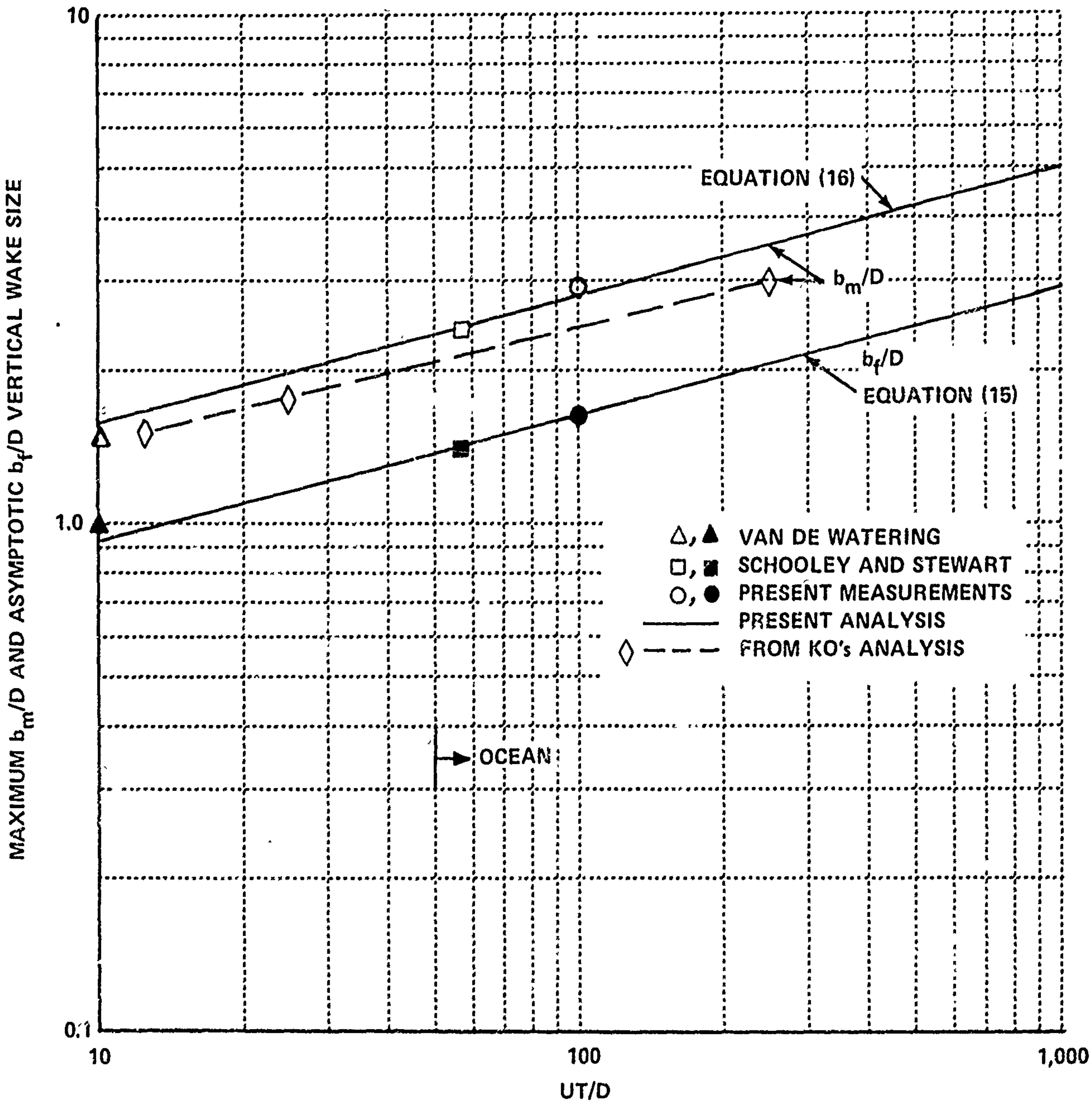

Figure 11 CORRELATION OF MAXIMUM AND ASYMPTOTIC VERTICAL WAKE SIZES

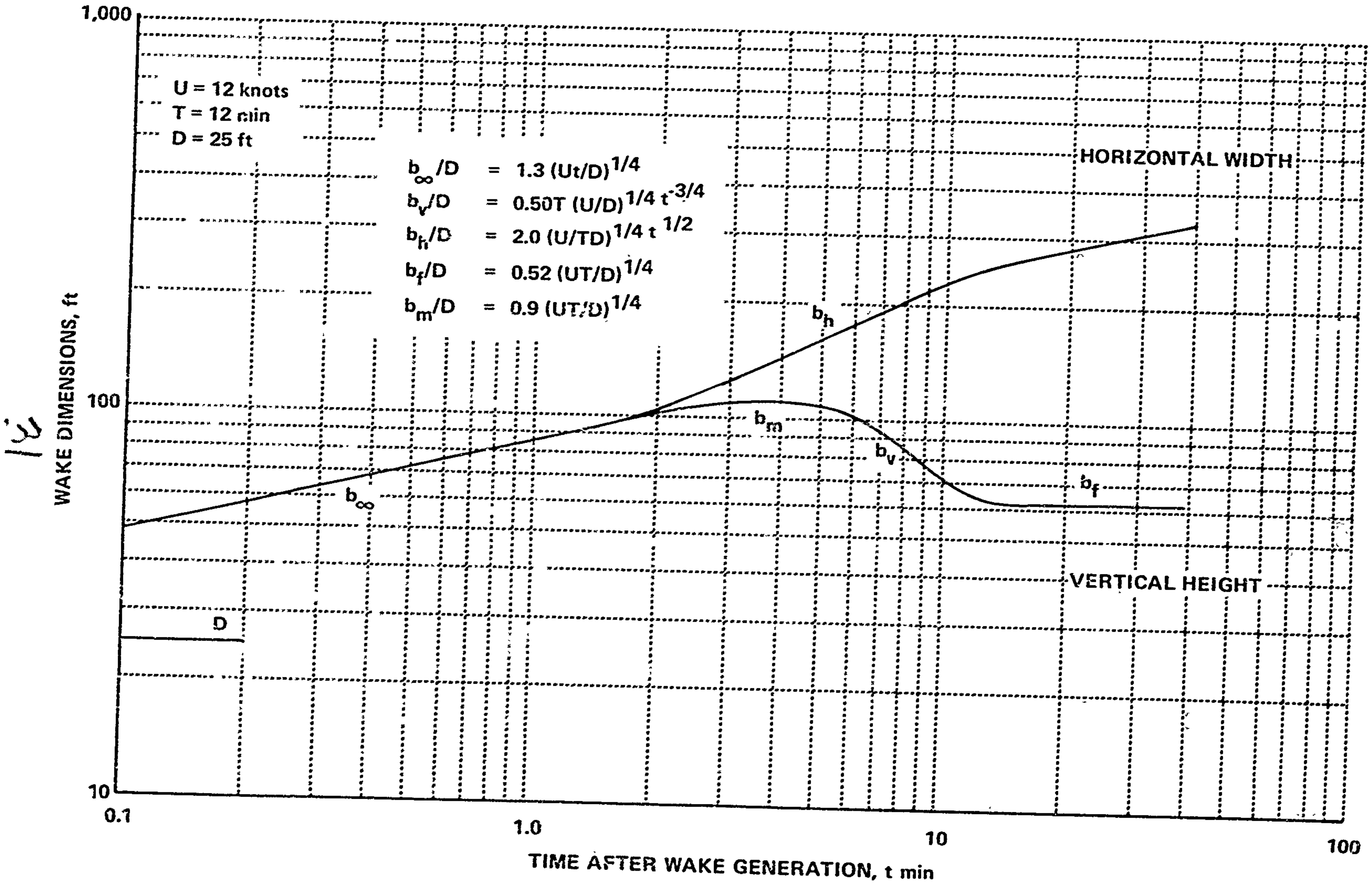

Figure 12 ESTIMATES OF WAKE SIZE, FULL SCALE OCEAN